Further Studies of Harmonic Gradient Method for Supersonic Aeroelastic Applications

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Recent developments in the applications of the harmonic gradient method (HGM) to various lifting surfaces with/without control surfaces are described. Our objective is to validate the acceleration-potential version of HGM, also known as the ZONA51 code, with available measured data and existing methods, which include the constant pressure panel method, the pressure mode method, and the piston theory. Unsteady supersonic aerodynamics over a leading-edge flap of an F-18 wing and a trailing-edge flap of a British Aerospace Corporation vertical fin are studied. Measured pressure jumps along the flap hinge lines are captured by the ZONA51 code whereas other methods fail to do so. A supersonic flutter analysis is performed for four different wing planforms; these include a 45-deg swept wing, a NASA 70-deg delta wing, a National Aerospace Plane (NASP)-type wing body, and the active flexible wing (AFW). For all cases considered, it is found that the present method yields favorable flutter trends which follow closely with those measured. Conservative flutter boundaries are obtained in almost all cases, in contrast to the predicted results of other existing methods. Finally, the AFW with fuselage and wing-tip ballast store is conveniently modeled and computed by the ZONA51 code resulting in a reasonable flutter boundary. It is believed that the ZONA51 code with its robust structure along with the present validation effort will be upheld as an integral part of matured aeroelastic technology.

Introduction

IT has been nearly six years since the first publication of the harmonic gradient method (HGM).[1] A series of work related to unsteady supersonic aerodynamics for wings and bodies and for wing-body combinations has been gradually developed within this period.[2-9] Meanwhile, further efforts have been made to improve the HGM and to validate its computer program, known as the ZONA51 code, with existing methods and available measured data.

Although the ZONA51 code has been adopted by a number of aerospace industries since 1984, few flutter results were released in the public domain due to company proprietorship. However, our recent collaboration with several ZONA51 users do not have such proprietory restrictions. In addition, some supersonic flutter results appear to be currently available in the open literature. It is the support from these sources that renders the present validation effort possible.

The purpose of the present paper is twofold. First, the improved version of the HGM, based on an acceleration potential formulation, will be briefly described. Next, applications of this improved version of HGM (or the ZONA51C code) to various wing planforms with or without control surface will be presented.

For control surface study, unsteady pressures for two wing planforms will be presented. Our computed results are compared with those obtained in Ref. 10 for a CF-18 wing with leading-edge flap oscillation and in Ref. 11 for a British Aerospace Corporation (BAC) vertical-fin planform with trailing-edge flap oscillation.

For flutter analysis study, flutter boundaries for four types of wing planform will be presented. Those include 1) a 45-deg swept wing tested by the Air Force Wright Aeronautical Laboratories (AFWAL) group,[12] 2) a NASA 70-deg delta wing,[13] 3) the NASP-type delta wing model mounted on a body,[13-15] and 4) the active flexible wing (AFW) currently being tested in the NASA Langley transonic dynamics tunnel.[16] Apart from the available measured data for validation, existing methods for comparison include those by the constant pressure panel method (CPM) of ASTROS (a newly developed multidisciplinary structural design code), Cunningham's pressure mode method, and the piston theory.

Acceleration-Potential Approach

The HGM is based on the velocity potential model whereby the integral solution for the oscillatory potential is obtained as

$$\phi = -\frac{1}{2\pi}\frac{\partial}{\partial n}\iint_A \Delta\phi(x,y,z)H(\xi,\eta,\zeta)\,\mathrm{d}x\,\mathrm{d}y \tag{1}$$

The supersonic kernel function H reads as

$$H(\xi,\eta,\zeta) = \frac{\cos\nu R}{R}e^{-i\nu M\xi} \tag{2}$$

and $\Delta\phi$ is the doublet solution to be sought, where $\nu = Mk/\beta$, k is the reduced frequency ($=\omega L/u_\infty$), and R is defined as

$$R = [\xi^2 - \eta^2 - \zeta^2]^{1/2} \tag{3}$$

where $\xi = x_0 - x, \eta = y_0 - y, \zeta = z_0 - z$.

The operator $(\partial/\partial n)$ can be expressed as

$$\frac{\partial}{\partial n} = -\ell_y\frac{\partial}{\partial y_0} - \ell_z\frac{\partial}{\partial z_0} = -\ell_y\frac{\partial}{\partial \eta} - \ell_z\frac{\partial}{\partial \zeta} \tag{4}$$

Received July 3, 1989; revision received Aug. 10, 1990; accepted for publication Aug. 10, 1990.

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where ℓ_y and ℓ_z are the direction cosines of the normal at any point on the planforms. Note that in Eq. (1), (x_0,y_0,z_0) and (x,y,z) represent the field-point and the sending-point locations, respectively. In particular, Eq. (1) is to be integrated over the area A, which encloses all sending points that originate from the wing panels as well as the wake region within the inverse Mach cone of influence (see Fig. 1).

Subsequently, it was realized that an acceleration potential formulation of HGM should be established in order to facilitate the computation scheme of wing-body combinations. In essence, this formulation will confine the computation domain to the wing planform only, and it still fully accounts for the unsteady wake effect. The integral potential in terms of the linearized pressure across the planform ΔC_p can be expressed as

$$\phi = \frac{\beta}{4\pi}\int_y \int_{x_{\mathrm{TE}}}^{x_{\mathrm{LE}}} \Delta C_p(x,y,z) K(x,\eta,\zeta)\,\mathrm{d}\xi\,\mathrm{d}y \tag{5}$$

where the kernal function K reads

$$K = -\left\{\int_X^{\xi} e^{-(ik\lambda/\beta)}\frac{\partial H}{\partial n}\,\mathrm{d}\lambda\right\} e^{-ik\beta\xi} \tag{6}$$

where λ is the dummy variable and $X = x_0 - \sqrt{(y_0-y)^2+(z-z_0)^2} \equiv x_0 - r$.

The unknown ΔC_p is related to the acceleration potential, as

$$\Delta C_p \equiv -2\left[\frac{1}{\beta}\frac{\partial}{\partial x} + ik\right]\Delta\phi \tag{7}$$

Clearly, as assured by the principle of kernel function formulation, the wake region as shown in Fig. 1, whether it is extended from the subsonic or supersonic trailing edges, can be excluded from the computation domain altogether.[17,18]

When integrated by parts, Eq. (1) can be recast into the following:

$$\phi = \frac{-1}{2\pi}\frac{\partial}{\partial n}\iint_A \frac{\partial}{\partial x}(\Delta\phi e^{-i\nu Mx}) S(\xi,r)\,\mathrm{d}A \tag{8}$$

where the kernel function S is expressed as

$$S = -\frac{\partial}{\partial n}\int_A^{\xi}\left(\frac{\cos\nu R}{R}\right)\mathrm{d}\lambda$$

Here, the HGM is introduced to Eq. (8) in order to evaluate the unknown $\Delta\phi$ in terms of the complex constants a_{ij}:

$$\frac{\partial}{\partial x}(\Delta\phi e^{-i\nu Mx}) = a_{ij} e^{-i\nu Mx} \tag{9}$$

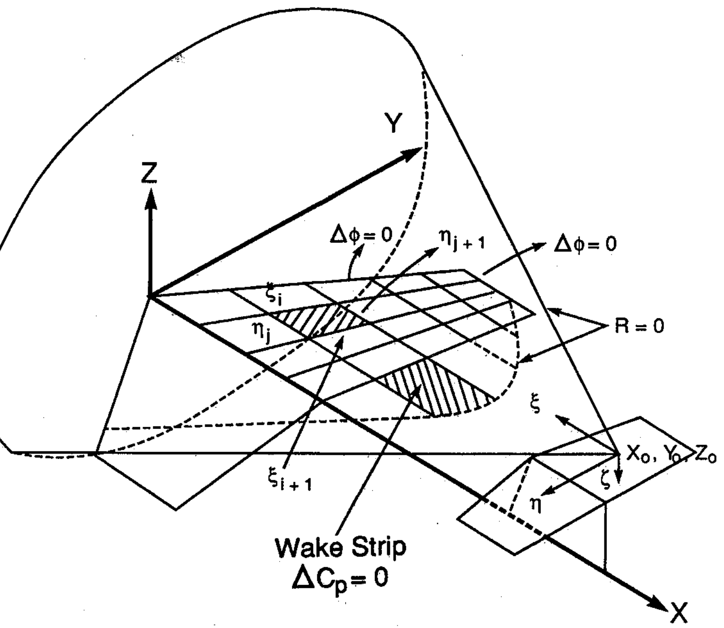


Fig. 1 Domain of influence and panel arrangement.

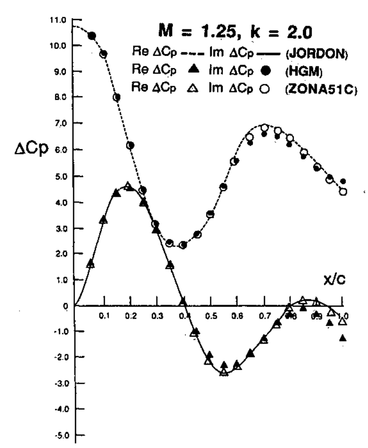


Fig. 2 Comparison of computed unsteady pressures for a plunging flat plate at $M = 1.25$ and $k = 2.0$.

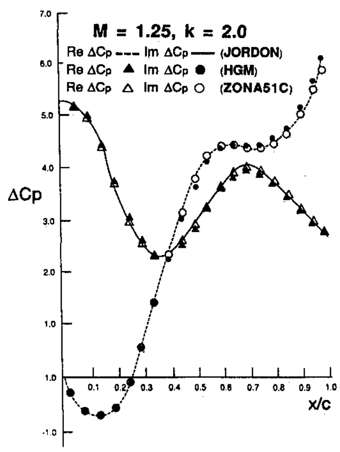


Fig. 3 Comparison of computed unsteady pressures for a pitching flat plate about the leading edge at $M = 1.25$ and $k = 2.0$.

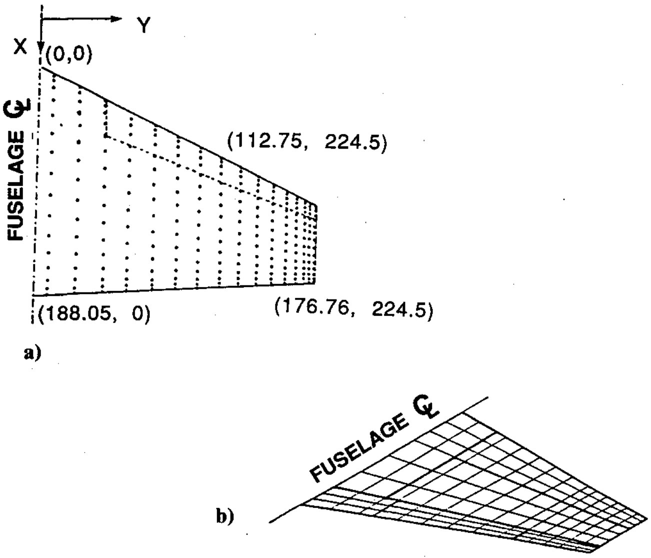


Fig. 4 Aerodynamic modeling of F-18 wing.

The counterpart of this unknown in the acceleration-potential formulation, Eqs. (5) and (6), is equivalently ΔC_p, which is expressed by $\mathrm{Re}(\Delta C_p) + i\,\mathrm{Im}(\Delta C_p)$.

In Figs. 2 and 3, computed results of $\mathrm{Re}(\Delta C_p)$ and $\mathrm{Im}(\Delta C_p)$ of a flat plate undergoing plunging and pitching motion about the leading edge are presented. It can be seen that the results computed by ZONA51C (acceleration-potential version of HGM) are in better agreement with Jordan's exact solution[19] than that of HGM (based on velocity potential). The close agreement with Jordan's[19] result near the trailing edge is attributed to an exact treatment of linearized pressure in the acceleration-potential formulation of ZONA51C. Computed results obtained by HGM and CPM of Appa[20] show similar degrees of discrepancies for the plunging case (Fig. 2) near the trailing edge.

Recent works of Lottati and Nissim[21] and Appa[20] are based on the acceleration-potential type of kernel functions approach. A close examination of the work of these authors reveals that their evaluation of the nonplanar integral is a rather approximate one. Both planar and nonplanar integrals in ZONA51C are obtained analytically. Numerical techniques used to solve these integrals are similar to those used in the doublet lattice method (DLM) by Rodden et al.[22] It should be noted that in Ref. 27, Cunningham also extended the analytical treatment on the kernel function singularities in his supersonic work including the 3/2 singularity along the Mach hyperbola in the nonplanar kernel functions. By constrast, the evaluation of the nonplanar kernel in Refs. 20 and 21 used a finite difference scheme to obtain numeric gradient of the planar kernel. In some cases, such a scheme would be inadequate. For example, when solving the nonplanar kernel for two nearly coplanar planforms in tandem, numerical inaccuracy will result if the departure height is small and is comparable with the difference step size Δr. One such typical configuration can be found in Fig. 2 of Ref. 22, which could be used as a working example for future method verifications.

Control Surface Studies

In this section, unsteady pressure distributions computed by ZONA51C for two different wing planforms with oscillating flaps are presented.

CF-18 Wing with Leading-Edge Flap

At the National Aeronautical Establishment (NAE) Canada, Lee has performed a comparative study[10] on computed pressures for an oscillating leading-edge flap on the F-18 wing at 58.8% span and at $M = 1.1$ and reduced frequency $k = 4.0$

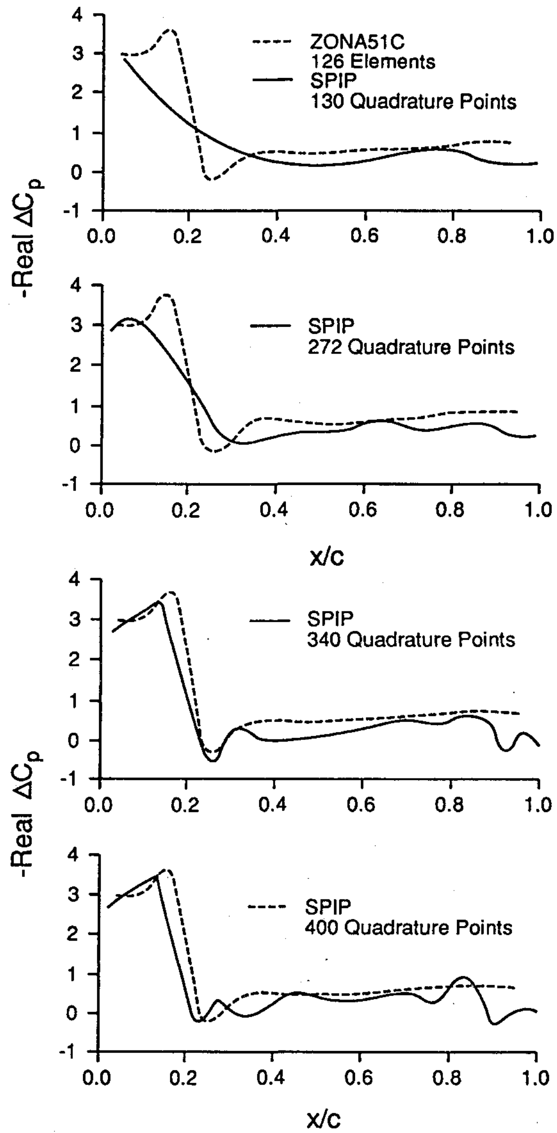


Fig. 5 Effects of panels on computed pressures for an F-18 wing with an oscillating leading-edge flap at 58.8% span, $M = 1.1$ and $k = 4.0$.

(see Fig. 4). Two computer codes are used: the ZONA51C code and the SPIP code.[23]

In Fig. 5, computed results show that the expected pressure jump at the hinge line is well predicted by ZONA51C using 126 panels. The results obtained by the SPIP code, however, depend on the number of quadrature points chosen where no pressure jump across the hinge line is seen until more quadrature points are used.

British Aerospace Corporation Fin with Trailing-Edge Flap

Under contract with BAC, oscillatory pressure measurements were made on a vertical-fin planform with a trailing flap model at the National Aerospace Laboratory (NLR), The Netherlands.[11] The fin planform geometry is given in Ref. 11. According to this geometry, the fin planform is subdivided into four subsystems in which the trailing-edge flap is represented by the subsystem 2 as shown in Fig. 6. The subsystem division is a convenient option provided by ZONA51C. To represent the trailing-edge flap oscillation, a no-motion condition is imposed on subsystems 1, 3, and 4, and subsystem 2 is specified to pitch about its own leading edge, which is really the hinge line.

Twelve panels are assigned to each chordwise strip in which denser grids are placed ahead of the hinge line.

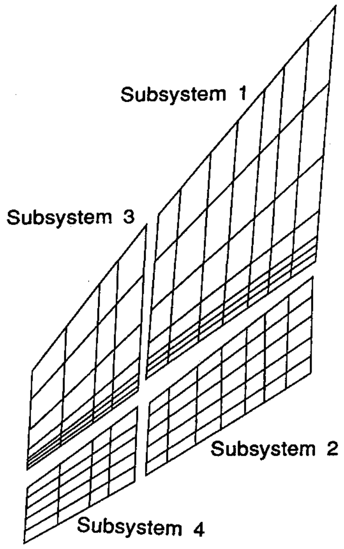


Fig. 6 Aerodynamic and subsystem modeling of the BAC vertical-fin planform.

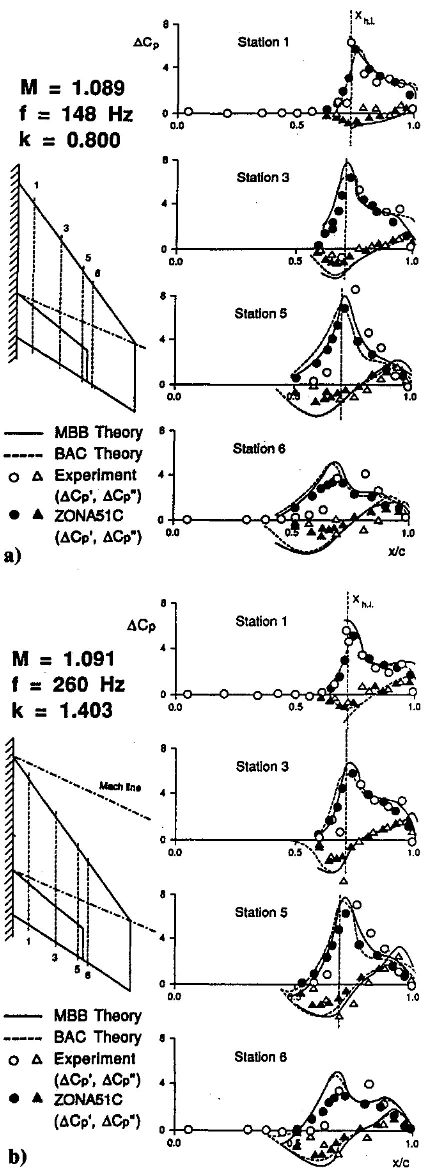


Fig. 7 Comparison with MBB theory, BAC theory, and measured data for a BAC vertical-fin planform with oscillating flap (subsonic trailing edges: case A and case B).

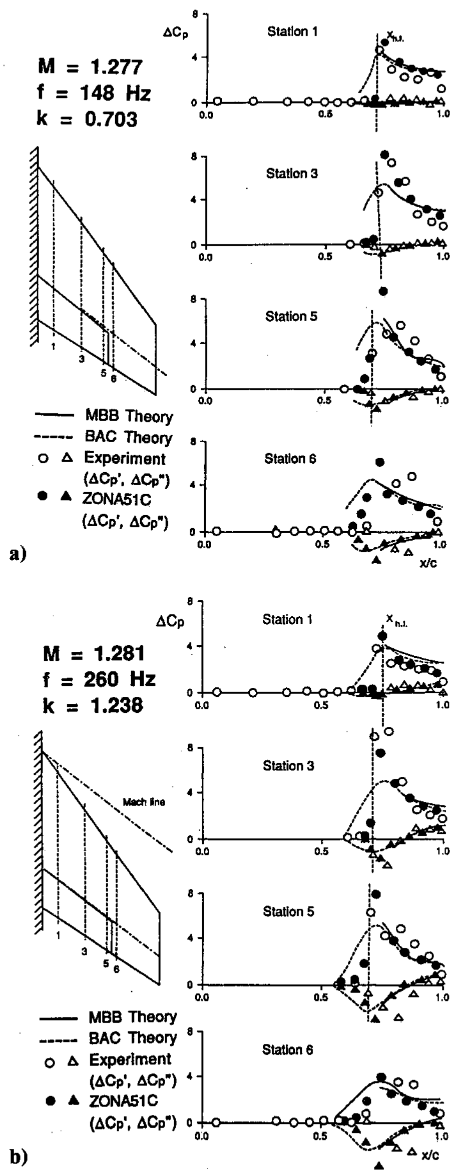


Fig. 8 Comparison with MBB theory, BAC theory, and measured data for a BAC vertical-fin planform with oscillating flap (supersonic trailing edges: case C and case D).

Figure 7 presents cases with a subsonic hinge line and subsonic trailing edge. Hence, the flap movement would influence the flow region upstream of the hinge line up to the Mach line. The Kutta condition at the trailing edge and the logarithmically singular pressure behavior in $\Delta C_p'$ about the hinge line are ascertained in this type of flow situation. It can be seen that all theories and ZONA51C result in good correlation in $\Delta C_p'$ with the measured data. However, better agreement is found between results of ZONA51C and the measured data for the out-of-phase $\Delta C_p''$.

Figure 8 shows cases with a nearly sonic hinge line and supersonic trailing edge. Little flow influence can be obtained by the flap motion ahead of the hinge line, across which pressure jump is expected to occur. Computed results of ZONA51C confirm with the measured data in these pressure jumps for all cases at stations 1, 3, and 5. Because of the limitation in the BAC/MBB methods, both fail to yield the results with pressure jump at the hinge line for all cases considered.

Studies of Wing Flutter

A collection of flutter results for several wing planforms are presented in this section. For flutter point prediction, the *V-g* method is used for all cases considered with the exception of the AFW case, which employs the root-locus plot method.

45-Deg Swept Wing Tested by AFWAL

An investigation has been conducted by the group in Ref. 12 to evaluate the capability of FASTOP[23] and ASTROS[24] codes by applying them to a 45-deg swept wing (Fig. 9). In Fig. 10, convergence studies on the number of panels used in ASTROS and ZONA51C for flutter speed (in knots equivalent air-speed) and flutter frequency are presented. Note that it is the CPM code[20] in ASTROS that provides the unsteady aerodynamics for the flutter analysis. It can be seen that the flutter results obtained by ZONA51C are still conservative but in better correlation to the experimental flutter point than those by CPM in both flutter speed and frequency. In obtaining the flutter results, three modes (solid dots) and four modes (solid triangle) supplied by NASTRAN are used for inputs to the ZONA51C code.

NASA 70-Deg Delta Wing

Flutter studies on a series of low-aspect-ratio delta planforms from subsonic to supersonic Mach numbers have been performed earlier at the NASA Langley Research Center.[13] Recent work performed there has supplied further computed results to correlate with the experimental data.[14]

To validate the ZONA51C code, a 70-deg swept delta wing (model 1A) in this series is selected for comparison with the existing flutter results. The delta planform is subdivided into 100 panels as shown in Fig. 11. According to Ref. 13, four modes are used in the present flutter analysis.

A set of flutter points were obtained for seven Mach numbers ($M = 1.01$, 1.19, 1.30, 1.64, 2.0, 2.25, and 3.0) using ZONA51C. Throughout the supersonic Mach number range considered, ZONA51C appears to yield the best correlation with experimental data among all supersonic methods shown here; these include FAST,[23] ACUNN,[27] piston theory (second order),[25] and CAP-TSD[29,30] (Figs. 12 and 13). Overall, ZONA51C predicts slightly lower values in the flutter frequency ratio ω_f/ω_2 (ω_2 is the natural frequency of mode 2) than the obtained experimental data.

NASP-Type Wing Body

Flutter characteristic of a 70-deg swept delta mounted on a body of revolution has been studied by Pototzky et al.[15] As shown in Fig. 14, the body is assumed to be rigid and flexible for Figs. 14a and 14b, respectively. Seven modes on the wing are used for Fig. 14a, whereas three free-free modes on the body and four modes on the wing are used for Fig. 14b. All wing modes are selected from the dominant flutter modes in Ref. 13.

Cases at three Mach numbers ($M = 1.2$, 2.5, 3.0) are computed as shown in Figs. 14a and 14b for the rigid body and the flexible body cases, respectively. Three supersonic codes are used for computations; these are indicated by PISTON (piston theory, e.g., Refs. 25 and 26), ACUNN,[27,28] and ZONA (actually ZONA51C). In Fig. 14a, it is seen that ZONA compared well with those of PISTON at two higher Mach numbers, 2.5 and 3.0, and with ACUNN at $M = 1.2$, whereas ACUNN over-

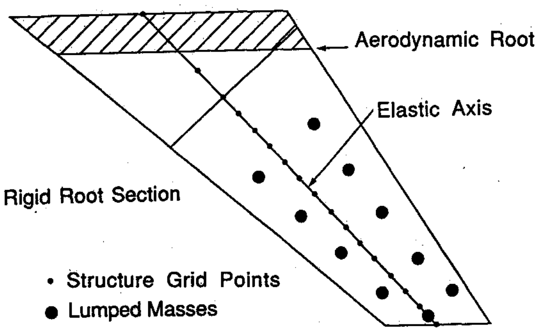


Fig. 9 AFWAL wing flutter model with lump mass NASTRAN representation.

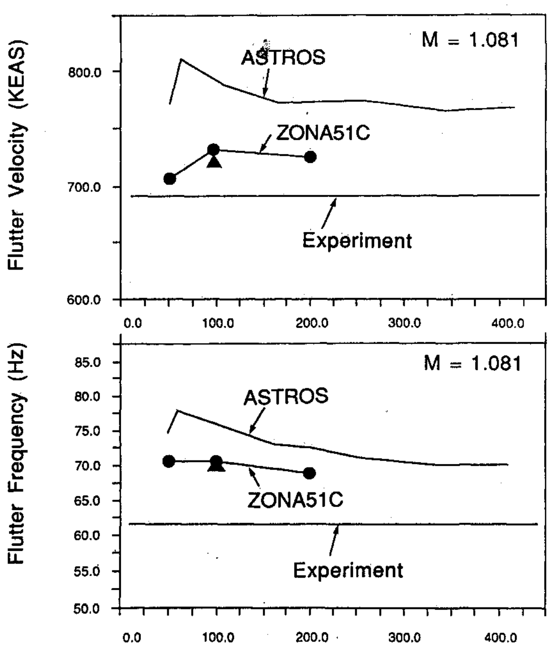


Fig. 10 Effects of panel numbers on flutter speed and frequency.

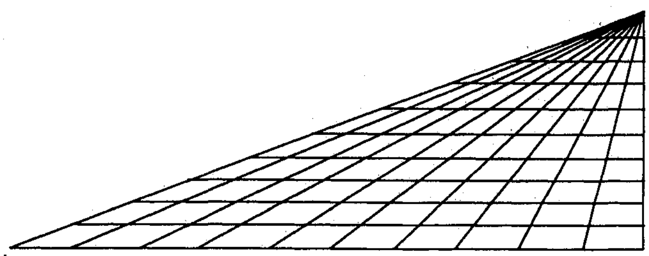


Fig. 11 Paneling scheme for a 70-deg delta wing.

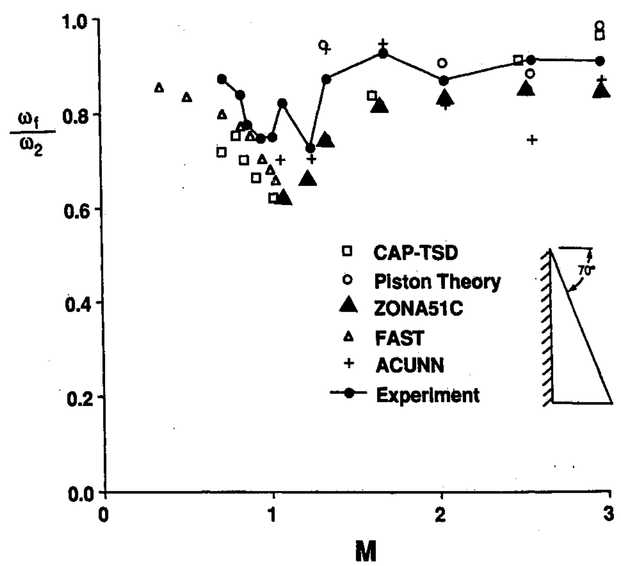


Fig. 12 Computed and measured flutter speeds vs Mach number.

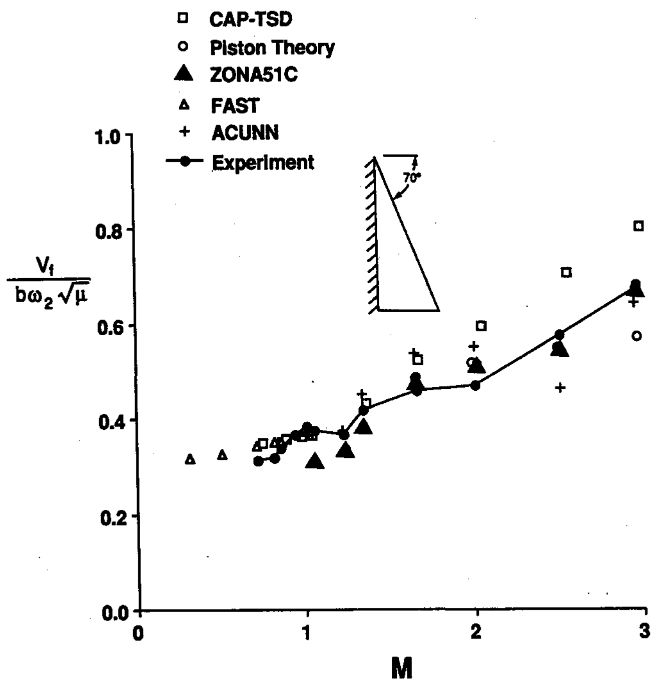
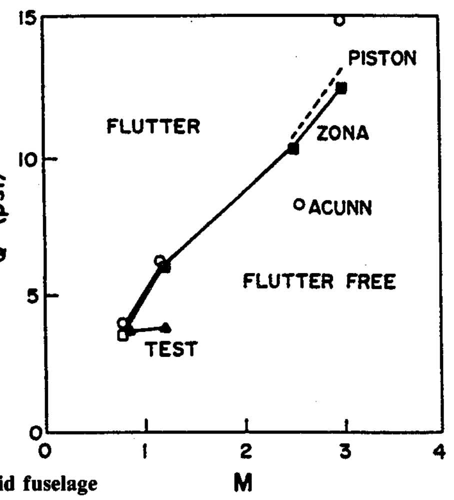
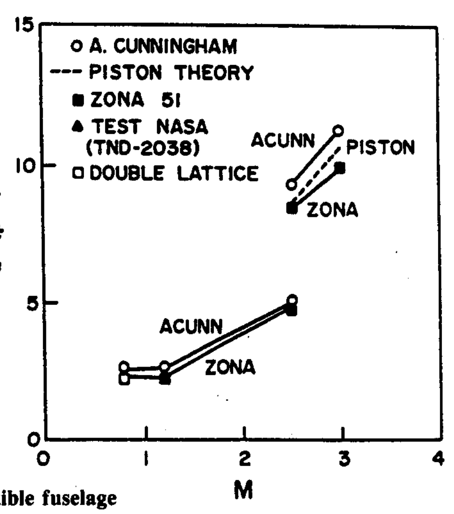


Fig. 13 Comparison of ratios of flutter frequency vs Mach number.



a) Rigid fuselage



b) Flexible fuselage

Fig. 14 Flutter boundaries for NASP-type wing-body configuration.

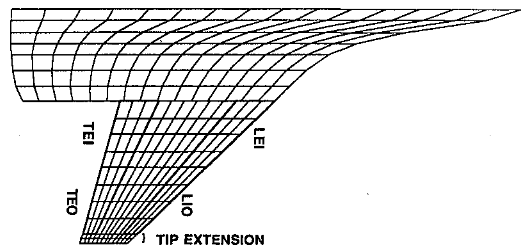


Fig. 15 Aerodynamic representation of AFW model.

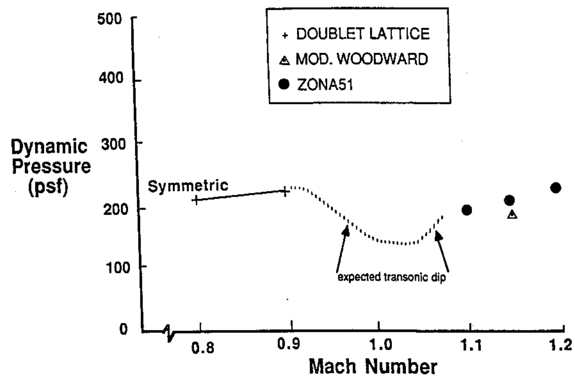


Fig. 16 Predicted flutter boundaries with Mach number.

estimates and underestimates these values at the higher Mach numbers. On the other hand, the test results by Hanson and Levey,[13] shown from $M = 0.8$ to 1.2, are obtained under a different condition where the wing under consideration is mounted on a flat wind-tunnel wall. The verification of the test data therefore awaits further generation of computed results.

Flutter points obtained by three codes agree well in Fig. 14b for the flexible body case. Note that ZONA generated consistently the most conservative boundary. All codes predict a jump in the flutter boundaries at $M = 2.5$. Presumably, it is caused by the mechanism of flutter-mode switching at this Mach number.

Several simplifications are noted for these calculations. First, all dynamic pressures of flutter Q (in psi) are computed based on a fixed speed of sound, namely, $a = 1000$ ft/s. Second, the wing-body effects are included only structurally but not aerodynamically in these studies, since the three codes used are confined to treatments of lifting surfaces only.

Active Flexible Wing Model

The AFW takes aeroelastic advantage of reduced structural weight and the corresponding increased flexibility of the wing to achieve high maneuver capability with its large multiple leading/trailing-edge control surfaces through the use of active controls and maintains satisfactory stability characteristics and safe load limits. As described in Ref. 16, several computer codes were used to generate the unsteady aerodynamics for AFW including the use of ZONA51C. Since NASA Langley's adoption of the ZONA51C code, further modification of it has been made in adding a general-purpose surface interpolation option in that the code can easily handle mode shape data from finite element programs using plate-type elements. This surface interpolation employs the surface plane method described by Harder and Desmarias in Ref. 31. Eight different spline surfaces are used to define the structural deflections: fuselage, sting, wing box, four control surfaces, and tip ballast.

The aerodynamic paneling to model lifting surfaces is shown in Fig. 15. The fuselage for the purpose of this analysis was also simplified to lifting surface aerodynamics. Again, in an attempt to simplify the aerodynamic representation, the tip ballast was treated as a 2.88-in. extension of the wing. Ten symmetric flexible modes were used in the analysis, and generalized aerodynamic forces were generated at 12 reduced frequencies, which ranged from 0 to 2. There were 225 panels used in the aerodynamic model, which was divided up into subsystems and lifting surfaces. A flutter analysis was performed using the ISAC DYNARES code described in Ref. 32. This code generates root locus or eigenvalue variation on the complex plane from a set of first-order flutter equations at a constant velocity as dynamic pressure is varied incrementally. To permit a first-order representation of the flutter equations, the unsteady aerodynamics in terms of reduced frequencies were fitted to s-plane rational function approximations. Figure 16 shows the computed flutter boundary vs Mach number. Doublet lattice code is used to obtain the subsonic aerodynamic flutter result, and, aside from three ZONA51 data points (1.10, 1.15, and 1.20), the other supersonic data point at 1.15 is obtained by a modified Woodward code from Rockwell.[33] The flutter boundary using ZONA51 aerodynamics is slightly higher than that of the modified Woodward code because in the latter the tip ballast store was modeled somewhat differently. It was represented by a much larger lifting surface extending beyond the leading edge and behind the trailing edge of the wing.

Conclusions

The developed unsteady supersonic program (ZONA51C code) has been carefully validated through the studies in control surface aerodynamics for two wing/flap configurations and in flutter boundary predictions for four different wing planforms. From the present studies, the following conclusions can be drawn. For control surface aerodynamics, the present method captures the hinge line pressure jump behaviors for an oscillating leading-edge flap of the F-18 wing and an oscillating trailing-edge flap of the BAC vertical-fin planform. Present computed results predict the unsteady pressure jumps as measured by the experiment in the latter case, whereas other existing methods fail to do so.

For flutter boundary predictions, four different cases are described in order.

1) 45-deg swept wing flutter: the flutter results obtained by ZONA51C are still conservative in better correlation with the measured data than those obtained by CPM in both flutter speed and frequency.

2) NASA's 70-deg delta wing flutter: the present method appears to yield the best correlation with the measured data among all existing supersonic methods.

3) NASP-type wing flutter: the present method predicts all correct flutter trends, and it generates consistently the most conservative flutter boundaries for both the rigid body and the flexible body cases.

4) AFW flutter: ZONA51C is a flexible program which can be modeled to represent wing-body aerodynamics resulting in a reasonable predicted flutter boundary for the AFW with a fuselage and a wing-tip ballast store.

The structure of the ZONA51C code has the ease of application in which its defined subsystem, modal input options, and optimized paneling scheme all serve as merits over other methods. It is believed that the present validation effort will uphold ZONA51C as an integral part of the matured aeroelastic technology.

Acknowledgments

The authors would like to thank Carson Yates and Thomas Noll of the NASA Langley Research Center for initiating the ZONA51 activity there and providing us with the AFW material, B. H. K. Lee of NAE/Canada for providing the F-18 wing results, and H. Schmid of MBB for providing the necessary information on the BAC fin/flap configuration in support of the present validation effort.

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